Addendum to:

Distributed Maximum Likelihood Sensor Network Localization


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With this addendum, we would like to add a few extra details and comments on a few points that we feel important, regarding our published paper [1].

**Point 1.** For completeness, we feel that we need to mention that the constraints \( \delta_{i,j} \geq 0 \) in (6b), \( \epsilon_{i,k} \geq 0 \) in (6c), \( Y \succeq 0 \) in (6d), and \( \gamma_{i,j} \geq 0 \) in (19c) are redundant. The reasons of this is that they are implied by the LMI constraints before them (via Schur complement). The reason we left them in the paper is for clarity of exposition.

**Point 2.** There is a typo in condition (26). The correct condition is

\[
L_i(y_{i}^{(t) + 1}, p_i^{(t) + 1}, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) - L_i(y_{i}^{(t)}, p_i, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) \leq \varepsilon, \quad \text{for all } y_{i,j}^{t} \in Z_{i,j}, p_i \in P_i.
\]

Here is the explanation. Consider the convex constrained optimization problem

\[
\min_{x \in X} f(x)
\]

then an optimizer \( x^\star \) is the one for which,

\[
f(x) - f(x^\star) \geq 0, \forall x \in X.
\]

If we want to solve the optimization problem in a feasible way up to an \( \varepsilon \) accuracy, then we need to find a point \( \hat{x} \in X \) for which

\[
f(x) - f(\hat{x}) \geq -\varepsilon, \forall x \in X.
\]

For the local problems the local optimization cost is

\[
L_i(y_{i}^{(t) + 1}, p_i^{(t) + 1}, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) := f_i(y_{i}^{(t)}, p_i) + \sum_{j \in N_i} [\lambda_{i,j}^{T} y_{i,j}^{(t)} + \frac{P}{2} \| y_{i,j}^{(t)} - z_{i,j} \|^2],
\]

and its \( \varepsilon \)-accurate and feasible solution is \( (y_{i}^{(t) + 1}, p_i^{(t) + 1}) \), which in light of the discussion before, implies

\[
L_i(y_{i}^{(t) + 1}, p_i^{(t) + 1}, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) - L_i(y_{i}^{(t)}, p_i, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) \geq -\varepsilon, \quad \text{for all } y_{i,j}^{t} \in Z_{i,j}, p_i \in P_i,
\]

or equivalently,

\[
L_i(y_{i}^{(t) + 1}, p_i^{(t) + 1}, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) - L_i(y_{i}^{(t)}, p_i, z_{i,j}^{(t)}, \lambda_{i}^{(t)}) \leq \varepsilon, \quad \text{for all } y_{i,j}^{t} \in Z_{i,j}, p_i \in P_i.
\]

which is (26) up to an unfortunate typo. Note that there is no need to require a lower bound.

**Point 3.** A few lines of explanation are in order for the sentence

*We note that \( p_i \) and \( z_{i,j} \) are not independent, but this will not be an issue.*

in Section VI.A.

Let us explain it. The distinction between the \( p \) and \( z \) (and then \( y \)) variables is purely formal, they are only a label to make our life easier (and the one of the reader) to follow our derivation. When we
optimize, we write down what the variables actually contain and consider the overlapping parts only once. That is, the reader should interpret the problem

\[
\begin{align*}
\text{minimize} & \quad f_i(y_i^i, p_i) + \sum_{j \in N_i} \left[ \lambda_{i,j}^{(l)} y_{i,j}^i + \frac{\rho}{2} \gamma_{i,j} \right] \\
\text{subject to} & \quad \left( \frac{1}{y_{i,j}^i - z_{i,j}^{(l)}} \right) \geq 0, \\
& \quad \gamma_{i,j} \geq 0 \quad \text{for all } (i, j) \in N_i,
\end{align*}
\]

by substituting,

\[
y_{i,j}^i = \left( [Y_{ii}, Y_{jj}, \delta_{i,j}, \delta_{i,j}, d_{i,j}, X_i^T, X_j^T]^T \right)^i, \quad p_i := \left( [\epsilon_i^T, e_i^T, Y_{ii}^i, x_i^T]^T \right)^i,
\]

which leads to

\[
\begin{align*}
\text{minimize} & \quad f_i \left( \left\{ \left[ \left( [Y_{ii}, Y_{jj}, \delta_{i,j}, \delta_{i,j}, d_{i,j}, X_i^T, X_j^T]^T \right)^i \right\} + \sum_{j \in N_i} \left[ \lambda_{i,j}^{(l)T} \left( [Y_{ii}, Y_{jj}, \delta_{i,j}, \delta_{i,j}, d_{i,j}, X_i^T, X_j^T]^T \right)^i + \frac{\rho}{2} \gamma_{i,j} \right] \\
\text{subject to} & \quad \left( \left( [Y_{ii}, Y_{jj}, \delta_{i,j}, \delta_{i,j}, d_{i,j}, X_i^T, X_j^T]^T \right)^i - z_{i,j}^{(l)} \right) \gamma_{i,j} I_{5+2D} \geq 0, \\
& \quad \gamma_{i,j} \geq 0 \quad \text{for all } (i, j) \in N_i.
\end{align*}
\]

That is to say: the fact that some variables are both in y and in p is irrelevant, as long as we don’t consider them as separate variables when we perform the optimization. So, this dependence is not an issue, as we wrote in the paper.

We thought that this purely formal substitution would have been easier to follow that carrying over all the variables, or have to impose extra equality constraints.

**Point 4.** For the extension of the work to dynamical settings, please refer to [2].

**References**
