A Moving Horizon Convex Relaxation for Mobile Sensor Network Localization

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Contributions in a nutshell

→ A moving horizon convex estimator that is based on a convex relaxation of the maximum a posteriori (MAP) formulation of the mobile network localization problem. (Extension of static convex relaxation to a dynamical ones.)

→ The relaxation is double: relaxation of the estimation problem within the window and relaxation of the term that summarizes the past information outside the estimation window (arrival cost).

→ Computational complexity comparable with the ones of extended and unscented Kalman filters; preliminary results indicate improved convergence speed and, when used together with a refinement, similar steady-state accuracy.

Step 1: MAP moving horizon

Formulation of the MAP problem (Gaussian noise assumption) in a fixed moving window. Approximation of the past info as arrival cost (EKF-based).

\[
\begin{align*}
\{S_{MAP,T}\}_\tau &\in \arg \min_{\{S\}_\tau} \sum_{\tau=0}^{T-1} \left( \sum_{(i,j)\in E} \|d_{i,j},r x_{i}, x_{j} - r_{i,j}\|_{2}^2 + 
\sum_{(i,k)\in E} \|d_{i,k},r a_{i}, a_{k} - a_{i,k}\|_{2}^2 \right) + 
\sum_{\tau=1}^{T-1} \left( \|S_{\tau+1} - \Phi_{\tau} S_{\tau}\|_{2}^2 + \|\text{vec}(S_{\tau} - S_{0})\|_{2}^2 \right)
\end{align*}
\]

where \( \tau_0 := \max\{\tau - T, 0\} \).

Step 2: Convex relaxation

The MAP problem is non-convex and NP-hard: introduction of the slack variable \( Y_{\tau} = X'_{\tau} X_{\tau} \) and relaxation of it as \( Y_{\tau} \succeq X'_{\tau} X_{\tau} \) (rank relaxation).

Similarly, \( \delta_{ij,r} = d_{ij,r}^2 \) and \( \epsilon_{i,k} = e_{i,k} \), and their relaxations \( \delta_{ij,r} \succeq d_{ij,r}^2 \) and \( \epsilon_{i,k} \succeq e_{i,k} \).

Define \( J(\{S\}_{\tau}, \{Y, \delta, \epsilon, d, e\}_{\tau}) := \)

\[
\begin{align*}
\sum_{\tau=0}^{T-1} \left( \sum_{(i,j)\in E} \|d_{i,j,r} x_{i} - r_{i,j}\|_{2}^2 + \sum_{(i,k)\in E} \|d_{i,k,r} a_{i} - a_{i,k}\|_{2}^2 \right) + 
\sum_{\tau=0}^{T-1} \left( \|S_{\tau+1} - \Phi_{\tau} S_{\tau}\|_{2}^2 + \|\text{vec}(S_{\tau} - S_{0})\|_{2}^2 \right)
\end{align*}
\]

subject to for \( \tau = \tau_0 + 1, \ldots, \tau \)

\[
\begin{align*}
Y_{\tau} + Y_{\tau+1} - 2Y_{\tau+1} &\in S_{\tau}, d_{ij,r} \succeq 0, \ \forall (i,j) \in E_{\tau} \cup \Omega_{\tau}^1 \nY_{\tau} - 2X'_{\tau} x_{\tau} + |a_{i},r|_{2}^2 &\in \epsilon_{i,k}, \ \forall (i,k) \in E_{\tau} \cup \Omega_{\tau}^2 \nY_{\tau} - X'_{\tau} x_{\tau} &\in \Omega_{\tau+1}^1 \nX_{\tau} &\in S_{\tau}, Y_{\tau} \in S_{\tau}^0
\end{align*}
\]


Simulation Results

Setting: \( D = 2, n = 30, m = 6, \Phi_{1} = (12, 0.1, \Phi_{0}, 1), \sigma_{ij} = \sigma_{i,k} = 0.05, \Sigma_{w} = \sigma_{w}^2 I, \Sigma_{v} = 5e^{-3} \), \( S_{0} = (0, 0, 0, 0)^T \), \( \Pi_{s_{0}} = 1/12 I_{1}, T = 5, R = 10. \) The number of range measurements per sensor node is limited to 3.

Metric: positioning root mean squared error (PRMSE) of the algorithms:

\[
\text{PRMSE} := \sqrt{\sum_{i=1}^{n} \sum_{v \in V} e_{i,v}^2 / R}
\]

Step 3: Edge-based convex relaxation

The rank relaxation and the arrival cost covariance \( \Pi_{s_{0}} \) couple all the nodes. To reduce the computational complexity: edge-based relaxations:

\[
\begin{align*}
\Pi_{s_{0}} \rightarrow \Pi_{s_{0}}^{\hat{\Omega}_{1}} \Pi_{s_{0}}^{\hat{\Omega}_{2}} \Pi_{s_{0}}^{\hat{\Omega}_{3}},
\end{align*}
\]

for all \( (i,j) \in E_{\tau} \)

\[
\begin{align*}
\min_{\{S\}_{\tau}, \{Y, \delta, \epsilon, d, e\}_{\tau}} J(\{S\}_{\tau}, \{Y, \delta, \epsilon, d, e\}_{\tau}),
\end{align*}
\]

subject to for \( \tau = \tau_0 + 1, \ldots, \tau \):

\[
\begin{align*}
Y_{\tau} &\in S_{\tau}, Y_{\tau} \in S_{\tau}^0
\end{align*}
\]

Extended Kalman filter
Unscented Kalman filter
Edge-based static convex relaxation of [1]
Edge-based moving h.
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